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Some Methods For Constructing Efficiency Balanced Designs

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SUMMARY

Some methods for constructing efficiency balanced designs using (i) circular designs and (ii) Balanced incomplete block designs have been discussed.

Keywords : Efficiency balanced designs; Circular designs; Balanced incomplete block designs and Efficiency factor.

Introduction

Several methods of construction of efficiency balanced designs are available in the literature (see: Calinski [I]; Puri and Nigam [8][9]; Williams [10]; Kageyama [6][7]; Dey and Singh [4]; Das and Ghosh [2]; Ghosh and Karmokar[5] etc.).

In this paper, some more methods are provided for constructing efficiency balanced designs from Circular designs (Das, [3]) and BIB designs.

2. *Method of Construction*

2.1 *Construction of efficiency balanced designs from circular designs.*

Construct a circular design with v treatments and block size k = 2. Choose p blocks form the b blocks of the circular design in such a way that the selected p blocks of a circular design do not contain any common treatment, that is, all the p blocks are disjoint, where $p = (v-1)/k = (v-1)/2$. Clearly v must be odd. Put these 2p treatments into one block and repeat this block (v – 3) times. Taking these (v – 3) blocks along with $\binom{v-1}{2}$ blocks, obtained from all the possible combinations of (v – 1) treatments taking two treatments together, and then augmenting the last treatment in each of $\binom{v-1}{2}$ blocks, an incomplete block design can be obtained. This incomplete block design is an efficiency balanced design with parameters $v' = v, b' = \binom{v}{2} - 2$

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$$r_i = 2v-5 \text{ for } i = 1, 2, \ldots, v-1,$$

$$= \binom{v-1}{2} \text{ for } i = v.$$

$$k_j = v-1 \text{ for } j = 1, 2, \ldots, v-3,$$

$$= 3, \text{ otherwise.}$$

Example 2.1. Let us construct a circular design with treatments v = 9 and block size 2. Select p = 4 disjoint blocks having treatments

(1, 2), (3, 4), (5, 6) and (7, 8) and put these 8 treatments in a block like (1, 2, 3, 4, 5, 6, 7, 8) and repeat this block 6 times. Other blocks of this design are

(1, 2, 9), (1, 3, 9), (1, 4, 9), (1, 5, 9), (1, 6, 9), (1, 7, 9), (1, 8, 9), (2, 3, 9), (2, 4, 9), (2, 5, 9), (2, 6, 9), (2, 7, 9), (2, 8, 9), (3, 4, 9), (3, 5, 9), (3, 6, 9), (3, 7, 9), (3, 8, 9), (4, 5, 9), (4, 6, 9), (4, 7, 9), (4, 8, 9), (5, 6, 9), (5, 7, 9), (5, 8, 9), (6, 7, 9), (6, 8, 9), (7, 8, 9).

The final design is an efficiency balanced design with parameters $v' = 9$, $b' = 34$, $r = 13$ for $i = 1, 2, 3, \ldots, 8$, $r = 28$, $k = 8$ for $j = 1, 2, 3, \ldots, 6$ and $k = 3$, otherwise, with efficiency factor, say, $E = 11/13$.

2.2. *Construction of efficiency balanced designs from BIB designs with* $k = 2$.

Let us consider a BIB design with the parameters $v, b = v(v-1)/2, r = (v-1), k = 2$ and $\lambda = 1$. Let p be an integer such that $kp = v - 1$ and choose p disjoint blocks from the b blocks of the above series of BIB designs. Since we are considering BIB design with $k = 2$ only hence v must be odd so as to get p as an integer. Put these 2p treatments into one block and repeat this block n times where $n = \lambda(k-1) = \lambda$. Taking n blocks along with r more blocks, obtained from $\binom{v}{2}$ blocks after deleting all those $\binom{2p}{2}$ blocks containing 2 treatments together of the 2p chosen treatments, an efficiency balanced design is obtained with parameters

$$v' = v, b = r+n, r' = r_i = 2 \text{ for } i = 1, 2, \ldots, v-1,$$

$$= r \text{ otherwise}$$

$$k' = k_j = v-1 \text{ for } j = 1, 2, \ldots, n,$$

$$= 2 \text{ otherwise with}$$

$$c = \lambda/2r(\lambda+n) \text{ and } E = c\Sigma s_m.$$

Example 2.2. Let us construct a BIB design with parameters $v = 9$, $b = 36$, $r = 8$, $k = 2$ and $\lambda = 1$. Now choose $p = 4$ disjoint blocks which contain treatments (1, 2), (3, 4), (5, 6), and (7, 8). Put these 8 treatments in one block and repeat this block once. The remaining 8 blocks are obtained after deleting 28 blocks which are (1, 9), (2, 9), (3, 9), (4, 9), (5, 9), (6, 9), (7, 9), (8, 9).

It is obvious for this series as $\lambda = 1$ and $k = 2$, n will be always one, together with $E = (k^2 - 1)/k^2$, i.e. $E = 3/4$.

2.3. *Construction of efficiency balanced design from BIB design with any k.*

Construct a BIB design with parameters v, $b = \binom{v}{k}$, $r = \binom{v-1}{k-1}$, $k, \lambda = \binom{v-2}{k-2}$.

Let p be an integer such that $kp = v - 1$ and choose p disjoint blocks from b blocks of the above series of BIB designs. Put these kp treatments into one block and repeat this block n times where $n = \lambda(k-1) - (v-1)\binom{v-3}{k-3}$. Taking these n blocks along with r more blocks, obtained from $b = \binom{v}{k}$ blocks after deleting all those $\binom{kp}{p}$ blocks containing k treatments of kp chosen treatments, an efficiency balanced design with parameters

$$v' = v, b' = r + n, k' = k_j = v - 1 \qquad \text{for } j = 1, 2, \ldots, n$$

$$= k \qquad \text{otherwise,}$$

$$r' = r_i = (\lambda + n) \qquad \text{for } i = 1, 2, \ldots, v-1,$$

$$= r \qquad \text{otherwise, and } c = \lambda / rk(\lambda + n) \text{ is obtained.}$$

Example 2.3. Consider a BIB design with parameters $v = 10$, $b = 120$, $r = 36$, $k = 3$ and $\lambda = 8$. Choose $p = 3$ which contain the treatments (1, 2, 3), (4, 5, 6), (7, 8, 9). Put these 8 treatments (1, 2, 3, 4, 5, 6, 7, 8) in one block and repeat this block 7 times. Other blocks are

(1, 2, 10), (1, 3, 10), (1, 4, 10), (1, 5, 10), (1, 6, 10), (1, 7, 10), (1, 8, 10), (1, 9, 10), (2, 3, 10), (2, 4, 10), (2, 5, 10), (2, 6, 10), (2, 7, 10), (2, 8, 10), (2, 9, 10), (3, 4, 10), (3, 5, 10), (3, 6, 10), (3, 7, 10), (3, 8, 10), (3, 9, 10), (4, 5, 10), (4, 6, 10), (4, 7, 10), (4, 8, 10), (4, 9, 10), (5, 6, 10), (5, 7, 10), (5, 8, 10), (5, 9, 10), (6, 7, 10), (6, 8, 10), (6, 9, 10), (7, 8, 10), (7, 9, 10), (8, 9, 10).

The parameters of the EB design are

$v' = 10, b' = 43, r' = r_i = 15$ for $i = 1, 2, \ldots, 9$

$= 36$ for $i = 10,$

$k' = k_j = 9$ for $j = 1, 2, \ldots, 7,$

$= 3$ otherwise, with $E = 38/45$.

Remarks.

The parameters of the circular and BIB design used for obtaining EB design along with efficiency factor, E, of EB design are given below

1.	v = 9 = b	r = k = 2				E = 0.8462
	v	b	r	k	λ	E
2.	9	36	8	2	1	0.75
3.	10	120	36	3	8	0.8444
4.	9	126	56	4	21	0.8958
5.	11	462	210	5	84	0.92
6.	13	1716	792	6	330	0.9352

From the above design it is clear that for k = 2, the efficiency factor, E, of the EB design obtained form circular design is larger than obtained from BIB design for the same treatments. It is also obvious as the block sizes of the BIB design increase the efficiency factor of EB design, obtained form the BIB design having the parameters v, $b = \binom{v}{k}$, $r = \binom{v-1}{k-1}$, k and $\lambda = \binom{v-2}{k-2}$ also increases.

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